

Interference of squeezed light produced by two independent optical parametric oscillators

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Received July 13, 1994; revised manuscript received January 27, 1995

The interference of the squeezed light produced by two independent, identical optical parametric oscillators below threshold is discussed in the context of semiclassical theory. It is shown that interference exists in the usual sense between the output fields from two optical parametric oscillators with coherent injected signals. The degree of squeezing and the intensity and noise fluctuation of the interference field are calculated in detail.

PACS numbers: 42.50.Dv, 42.50.Lc, 42.65.Ky. © 1995 Optical Society of America

1. INTRODUCTION

Squeezed states of light have become the subject of important investigations in contemporary quantum optics. It has been demonstrated that frequency down-conversion in an optical parametric oscillator (OPO) is a physical process for the generation of squeezed states.¹⁻⁴ Squeezed light has been employed for improvement of measurement precision beyond the vacuum-state limit in interferometry and spectroscopy.⁵⁻⁷ In practical applications the superposition and interference of two squeezed lights may often occur, so we study in detail the interference of two squeezed lights produced by two OPO's below threshold.

Ghosh *et al.* have discussed the interference between a signal and an idler photon produced in parametric downconversion.⁸ Belsley *et al.* have shown theoretically that the interference of independent broadband squeezed vacua can produce photon twin beams with perfect intensity correlation.⁹ Dodson and Vyas have discussed, with considerable sophistication and detail, the statistical features of the superposed field when the output light from a degenerate parametric oscillator is homodyned with a coherent local-oscillator field.¹⁰ Caves *et al.* have theoretically calculated the photon-number distribution for two-mode squeezed states with coherent amplitude.¹¹ Second-order interference has been successfully observed by Zou *et al.* in the superposition of signal photons from coherently pumped parametric downconverters without cavities.¹² To our knowledge, the interference of the squeezed light derived from two independent OPO's with nondegenerate parametric downconversion has not been discussed in the literature. Here we study this subject. First the basic equations of the OPO's are presented; then the average photon flux and the fluctuation in the spectrum of the interference field are analyzed. The results indicate that without the injected signal beams there is no interference in the usual sense between the squeezed-state lights produced by two independent OPO's pumped by beams derived from a common laser source. The intensity of the superposition field does not depend on the relative phase between the two overlapping squeezed beams. But, if a coher-

ent signal beam is split and then injected into the two OPO's, interference is present. The degree of squeezing of the interference field cannot be larger than that of the original overlapping squeezed fields.

2. BASIC EQUATION OF THE OPTICAL PARAMETRIC OSCILLATORS

A block diagram of the interference of two squeezed-state beams is shown in Fig. 1. M_1 and M_2 are 50% beam splitters, and M_3 and M_4 are perfectly reflecting mirrors. The squeezed lights generated in OPO1 and OPO2 through type 2 parametric downconversion are superposed through M_2 ; then the intensity of the resultant field is detected by detector D.

We use the semiclassical approach proposed by Fabre *et al.*¹³ to calculate the interference of the output fields from the OPO cavities and the field fluctuations. In this case, we describe the dynamics of small field fluctuations by linearizing the classical equations of motion in the vicinity of the stationary state. We consider these field fluctuations to be driven by the vacuum fluctuations entering the cavity through the coupling mirror.¹³ Treating the pump light as a classical field and neglecting the dissipation of the pump field and the detuning of the OPO cavities, we can write the C-number Langevin equation of the system as^{13,14}

$$\tau \dot{\alpha}_1 + (\gamma_1 + \gamma_1') \alpha_1 = g \epsilon_0 \alpha_2^* + \sqrt{2\gamma_1} \alpha_1^{in} + \sqrt{2\gamma_1'} c_1^{in}, \quad (1a)$$

$$\tau \dot{\alpha}_2 + (\gamma_2 + \gamma_2') \alpha_2 = g \epsilon_0 \alpha_1^* + \sqrt{2\gamma_2} \alpha_2^{in} + \sqrt{2\gamma_2'} c_2^{in}, \quad (1b)$$

where α_i ($i = 1, 2$) represent the intracavity signal and idler field amplitudes associated with their annihilation operators, ϵ_0 is proportional to the amplitude of the classical pump field, τ is the cavity round-trip time for the signal and the idler modes, g is the nonlinear coupling parameter that depends on the second-order nonlinear coefficient $\chi^{(2)}$ of the intracavity medium, γ_i is related to the losses of the output mirror of the cavity, and γ_i' is dependent on the absorption and scattering losses of the crystal

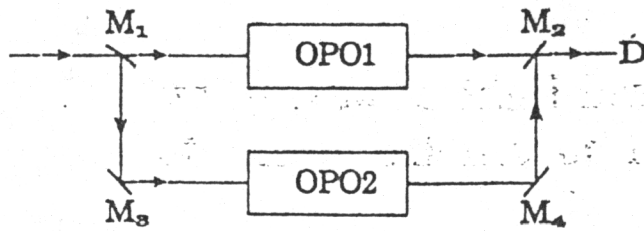


Fig. 1. Interference block diagram of two squeezed beams.

as well as on the losses of the other cavity mirrors. In the following calculation we assume that $\gamma_1 = \gamma_2 = \gamma$ and $\gamma_1' = \gamma_2' = \gamma'$, and we take $g, \gamma,$ and γ' as real quantities, i.e., the phase shift of the light field in the cavities is not considered. α_i^{in} and c_i^{in} are the field amplitudes of the injected noise from the output and the input mirrors, respectively.

The boundary condition obeyed by the output field is¹³

$$\alpha_i^{out} = \sqrt{2\gamma_i} \alpha_i - \alpha_i^{in}. \tag{2}$$

Taking $\alpha_i^{in} = c_i^{in} = 0$, we easily obtain the threshold pump power from Eqs. (1):

$$|\epsilon_0| = (\gamma + \gamma')/g. \tag{3}$$

The squeezing spectrum for the output field from an OPO may be found in the literature.¹⁵

3. SUPERPOSITION OF TWO SQUEEZED LIGHTS

If there is no injected signal from the input mirror of the OPO cavity, with significant loss for the signal and the idler resulting only from the output mirror, the cavity is called one sided.¹⁵ In this case we take $\gamma' = 0$, and Eqs. (1) become

$$\tau \dot{\alpha}_1 + \gamma \alpha_1 = g \epsilon_0 \alpha_2^* + \sqrt{2\gamma} \alpha_1^{in}, \tag{4a}$$

$$\tau \dot{\alpha}_2 + \gamma \alpha_2 = g \epsilon_0 \alpha_1^* + \sqrt{2\gamma} \alpha_2^{*in}. \tag{4b}$$

Solving Eqs. (2) and (4) in frequency space, we obtain the output field α_i^{out} from OPO1:

$$\alpha_1^{out}(\Omega) = \frac{A_1 \alpha_1^{in}(\Omega) + A_2 \alpha_2^{*in}(-\Omega)}{B}, \tag{5a}$$

$$\alpha_1^{*out}(-\Omega) = \frac{A_1 \alpha_1^{*in}(-\Omega) + A_2^* \alpha_2^{in}(\Omega)}{B}, \tag{5b}$$

$$\alpha_2^{out}(\Omega) = \frac{A_1 \alpha_2^{in}(\Omega) + A_2 \alpha_1^{*in}(-\Omega)}{B}, \tag{5c}$$

$$\alpha_2^{*out}(-\Omega) = \frac{A_1 \alpha_2^{*in}(-\Omega) + A_2^* \alpha_1^{in}(\Omega)}{B}, \tag{5d}$$

where

$$A_1 = \gamma^2 + \Omega^2 \tau^2 + g^2 |\epsilon_0|^2,$$

$$A_2 = 2g\epsilon_0\gamma,$$

$$B = (\gamma - i\Omega\tau)^2 - g^2 |\epsilon_0|^2.$$

The operators of the output-coupled mode for OPO1 are defined as¹⁶

$$d_1 = (\alpha_1 + \alpha_2)/\sqrt{2}, \tag{6a}$$

$$d_1^* = (\alpha_1^* + \alpha_2^*)/\sqrt{2}. \tag{6b}$$

When the input field is a vacuum state and thermal noise is neglected, the injected noise α_i^{in} obeys the following relations¹⁷:

$$[\alpha_i^{in}(\Omega) \alpha_j^{*in}(-\Omega')] = \delta_{ij} \delta(\Omega + \Omega'), \tag{7a}$$

$$[\alpha_i^{in}(\Omega) \alpha_j^{in}(\Omega')] = 0, \tag{7b}$$

$$[\alpha_i^{*in}(-\Omega) \alpha_j^{*in}(-\Omega')] = 0. \tag{7c}$$

We assume that the configurations for OPO1 and OPO2 are completely identical, so all expressions obtained for OPO1 [Eqs. (5)–(7)] can be used for OPO2 if we simply replace $\alpha_i^{out}, \alpha_i, \alpha_i^*, d_1, d_1^*$, and ϵ_0 with the corresponding variables for OPO2: $\beta_i^{out}, b_i, b_i^*, d_3, d_3^*$, and ϵ_0' .

The superposition field of the two output fields from OPO1 and OPO2 through beam splitter M_2 (Fig. 1) can be expressed as⁸

$$D = \frac{d_1 \exp[-i\omega_0(\tau_1 + \tau_0)] + id_3 \exp[-i\omega_0(\tau_3 + \tau_0)]}{\sqrt{2}}, \tag{8}$$

where $\tau_0, \tau_1,$ and τ_3 are, respectively, the propagation times of light from beam splitter M_2 to detector D and from OPO1 and OPO2 to M_2 .

The average photon flux at the detector is

$$R = \eta \langle D^* D \rangle. \tag{9}$$

Here η is the quantum efficiency of the detectors.

One can calculate R by using Eqs. (5)–(9):

$$\begin{aligned} R &= \frac{1}{2\pi} \eta \iint d\Omega d\Omega' \langle D^*(-\Omega) D(\Omega') \rangle \\ &= (1/2)\eta(d_1^* d_1) + (1/2)\eta(d_3^* d_3) \\ &= \frac{\eta g^2 |\epsilon_0|^2 \gamma}{\tau(\gamma^2 - g^2 |\epsilon_0|^2)} + \frac{\eta g^2 |\epsilon_0'|^2 \gamma}{\tau(\gamma^2 - g^2 |\epsilon_0'|^2)}. \end{aligned}$$

If $|\epsilon_0|^2 = |\epsilon_0'|^2$, one obtains

$$R = \frac{\eta g^2 |\epsilon_0|^2 \gamma}{\tau(\gamma^2 - g^2 |\epsilon_0|^2)}. \tag{10}$$

It is clear that there is no interference term that depends on the phase difference between the two squeezed lights d_1 and d_3 in Eq. (10), i.e., there is no interference effect in the usual sense. The total photon number of the resultant field is the sum of the photon numbers of the two overlapping squeezed lights.

4. INTERFERENCE OF TWO SQUEEZED LIGHTS

When the signal beam is simultaneously injected into the cavity through the input mirror with the pump light, the input and the output of the signal field are in two directions. This type of cavity is called a two-sided cavity.¹⁵ In this case the loss γ' cannot be designated zero; at the same time c_i^{in} can be replaced by $c_i^{in} + \epsilon_i$ in Eqs. (1), where ϵ_i denotes the amplitudes of the injected beams.

Solving Eqs. (1) in frequency space and using boundary condition (2), we obtain the output fields of the signal and idler lights:

(6a) $\alpha_1^{out}(\Omega) = \frac{A_2 \alpha_2^{*in}(-\Omega) + A_3 c_2^{*in}(-\Omega)}{B_1} + \frac{A_4 \alpha_1^{in}(\Omega) + A_5 c_1^{in}(\Omega)}{B_1} + \frac{A_3 \epsilon_2^* + A_5 \epsilon_1}{B_1}$, (11a)

(7a) $\alpha_1^{*out}(-\Omega) = \frac{A_2^* \alpha_2^{in}(\Omega) + A_3^* c_2^{in}(\Omega)}{B_1} + \frac{A_4 \alpha_1^{*in}(-\Omega) + A_5 c_1^{*in}(-\Omega)}{B_1} + \frac{A_3^* \epsilon_2 + A_5 \epsilon_1^*}{B_1}$, (11b)

(8) $\alpha_2^{out}(\Omega) = \frac{A_2 \alpha_1^{*in}(-\Omega) + A_3 c_1^{*in}(-\Omega)}{B_1} + \frac{A_4 \alpha_2^{in}(\Omega) + A_5 c_2^{in}(\Omega)}{B_1} + \frac{A_3 \epsilon_1^* + A_5 \epsilon_2}{B_1}$, (11c)

(9) $\alpha_2^{*out}(-\Omega) = \frac{A_2^* \alpha_1^{in}(\Omega) + A_3^* c_1^{in}(\Omega)}{B_1} + \frac{A_4 \alpha_2^{*in}(-\Omega) + A_5 c_2^{*in}(-\Omega)}{B_1} + \frac{A_3^* \epsilon_1 + A_5 \epsilon_2^*}{B_1}$, (11d)

where

$A_3 = 2g\epsilon_0\sqrt{\gamma\gamma'}$,
 $A_4 = \gamma^2 - (\gamma' - i\Omega\tau)^2 + g^2|\epsilon_0|^2$,
 $A_5 = 2[(\gamma + \gamma') - i\Omega\tau]\sqrt{\gamma\gamma'}$,
 $B_1 = [(\gamma + \gamma') - i\Omega\tau]^2 - g^2|\epsilon_0|^2$.

As mentioned above, we can obtain the same expression for OPO2 by replacing ϵ_1 and ϵ_2 with ϵ_3 and ϵ_4 .

It is assumed that the signal and the idler modes in the cavities have identical amplitudes and phases, i.e.,

$|e_1| = |e_2|, \quad \psi_1 = \psi_2$, (12a)

$|e_3| = |e_4|, \quad \psi_3 = \psi_4$. (12b)

Taking $|e_0| = |e_0'|$, we obtain the average photon number detected by detector D from Eqs. (6), (8), (11), and (12):

$R = \frac{1}{2\pi} \eta \iint d\Omega d\Omega' \langle D^*(-\Omega) D(\Omega') \rangle$
 $= \frac{\eta g^2 |\epsilon_0|^2 \gamma}{\tau [(\gamma + \gamma')^2 - g^2 |\epsilon_0|^2]} + \frac{\eta}{\tau^2} \{ 4\pi\gamma\gamma' (|e_1|^2 + |e_3|^2) + 8\pi\gamma\gamma'\eta |e_1| |e_3| \cos[\omega_0(\tau_1 - \tau_3) + (\psi_1 - \psi_3)] \}$, (13)

The integration in Eq. (13) is shown in detail in Appendix A. The third term on the right-hand of Eq. (13) is the interference term related to the phase difference $(\psi_1 - \psi_3)$ between the two signal fields injected into OPO1 and OPO2 and the difference between the propagation times $(\tau_1 - \tau_3)$ from the two OPO's to detector D. The interference effect depends on the intensity and the intracavity losses of the injected signal fields. The first and the second terms, respectively, are the

background intensities given by the pump and injected signal fields.

5. FLUCTUATION OF THE INTERFERENCE FIELD

The annihilation and creation operators of the interference field are defined as¹⁶

$D = (d_1 + id_3)/\sqrt{2}$, (14a)

$D = (d_1^* - id_3^*)/\sqrt{2}$. (14b)

The quadrature operators of the interference field are

$X_{D_+} = (D + D^*)/2$, (15a)

$X_{D_-} = (D - D^*)/2i$. (15b)

The uncertainty relation is

$\langle X_{D_-}, X_{D_+} \rangle = -1/2i$. (16)

This shows that the noise of the coherent state is 1/4. When the pump amplitudes and the cavity parameters of the two OPO's are assumed to be identical, the fluctuation in the spectrum of the interference field is obtained from Eqs. (11) and (15):

$S_{D_-}^{out}(\Omega) = \int_{-\infty}^{\infty} \langle X_{D_-}^{out}(\Omega), X_{D_-}^{out}(\Omega') \rangle d\Omega$
 $= -\frac{g\eta|\epsilon_0|\gamma}{4} \left\{ \frac{\cos\theta - \cos(\theta' + \pi) - 2}{[(\gamma + \gamma') - g|\epsilon_0|]^2 + \Omega^2\tau^2} + \frac{\cos\theta - \cos(\theta' + \pi) + 2}{[(\gamma + \gamma') + g|\epsilon_0|]^2 + \Omega^2\tau^2} \right\}$, (17)

where θ and θ' are the phases of pump fields ϵ_0 and ϵ_0' , respectively.

Setting $\cos\theta - \cos(\theta' + \pi) = 2$ and $\Omega = 0$, we obtain the maximum squeezing at the pump threshold [Eq. (3)]:

$S_{D_-}^{out,max}(0) = -(1/4) \frac{\gamma}{\gamma + \gamma'}$, (18)

For the one-sided cavity ($\gamma' = 0$) at the above-mentioned condition, $S_{D_-}^{out,max}(0) = -1/4$, and perfect squeezing is achieved. If

$P_0 = \cos\theta - \cos(\theta' + \pi) = \frac{4g|\epsilon_0|(\gamma + \gamma')}{(\gamma + \gamma')^2 + g|\epsilon_0|^2}$, (19)

we have $S_{D_-}^{out} = 0$, and the resultant field is not squeezed. Because the two squeezed lights are superposed through beam splitter M_2 , a π phase difference has been added between θ and θ' in the expression of $S_{D_-}^{out}(\Omega)$. The condition of maximum squeezing corresponds to $\theta = \theta' = 0$. We have assumed that OPO1 and OPO2 are pumped with a common laser source, so the condition of maximum squeezing is easily satisfied. The maximum degree of squeezing of the interference field is equal to that of a single overlapping field. When $2 \geq [\cos\theta - \cos(\theta' + \pi)] \geq P_0$ the interference field is squeezed ($S_{D_-}^{out} < 0$); otherwise the fluctuation of the resultant field is larger than the vacuum noise ($S_{D_-}^{out} > 0$);

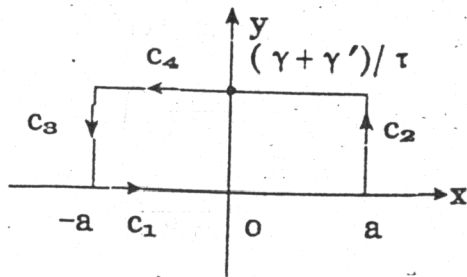


Fig. 2. Integrating loop.

i.e., the squeezing effect is not present. P_0 depends on $|\epsilon_0|$ and on the parameters of the cavity in Eq. (19).

6. CONCLUSIONS

The superposition and interference effects between the squeezed fields produced in two independent optical parametric oscillators through intracavity nondegenerate parametric downconversion have been discussed. The intensity distribution and the fluctuation in the spectrum of the resultant field have been calculated by the semiclassical method. We have shown that when the signal is not injected into the two OPO's, the interference pattern that depends on the phase difference between the two overlapping squeezed lights is not present. The output squeezed fields from the two OPO's interfere in the usual sense only when there are injected coherent signals. The fluctuation in the spectrum of the superposition field depends on the phases of the pump fields for the two OPO's but is not related to the injected signal fields, whereas the interference pattern depends only on the phase difference between the injected signal lights. Although the degree of squeezing of the resultant field cannot be larger than that of the single overlapping field, we can obtain a stronger intensity of the resultant squeezed-state light at the same noise level by controlling the phase difference of the pump fields and the phase difference of the signal fields between two cavities. With an injected coherent signal the OPO becomes a parametric amplifier, and the output field is a squeezed coherent state instead of a squeezed vacuum state. We assume that without the injected signals there is no phase correlation between the output squeezed fields from the two independent OPO's, so there is no interference pattern, whereas with coherent injected signals the phases of the output squeezed fields are defined by the phases of the injected fields. The calculation results can be used for both intracavity nondegenerate ($a_1 \neq a_2$) and degenerate ($a_1 = a_2$) parametric downconversion. Those concerned with the superposition of squeezed-state light might be interested in our discussion.

APPENDIX A

The two complex integrations in Eq. (13) are derived as follows:

$$\begin{aligned} & \int_{-\infty}^{\infty} \frac{d\Omega}{[(\gamma + \gamma') - i\Omega\tau]^2 - g^2|\epsilon_0|^2} \\ &= -\frac{1}{\tau^2} \int \frac{d\Omega}{[\Omega + i(\gamma + \gamma')/\tau]^2 + g^2|\epsilon_0|^2/\tau^2} \\ &= -\frac{1}{\tau^2} \int_{-x+i(\gamma+\gamma')/\tau}^{x+i(\gamma+\gamma')/\tau} \frac{dZ}{Z^2 + g^2|\epsilon_0|^2/\tau^2} \end{aligned} \quad (\text{A1})$$

Using the integrating loop in Fig. 2, we get

$$\begin{aligned} \text{Eq. (A1)} = & -\frac{1}{\tau^2} \left(\lim_{\sigma \rightarrow \infty} \int_{-\sigma}^{\sigma} \frac{dX}{X^2 + g^2|\epsilon_0|^2/\tau^2} \right. \\ & \left. - \int_c \frac{dZ}{Z^2 + g^2|\epsilon_0|^2/\tau^2} \right) \end{aligned} \quad (\text{A2})$$

Using the residue theorem and integrating, we get

$$\text{Eq. (A2)} = \begin{cases} -\pi/(\tau g|\epsilon_0|) & (> \text{pump threshold}) \\ 0 & (\leq \text{pump threshold}) \end{cases}$$

$$\begin{aligned} & \int_{-\infty}^{\infty} \frac{[(\gamma + \gamma') - i\Omega\tau]d\Omega}{[(\gamma + \gamma') - i\Omega\tau]^2 - g^2|\epsilon_0|^2} \\ &= \frac{i}{\tau} \int \frac{[\Omega + i(\gamma + \gamma')/\tau]d\Omega}{[\Omega + i(\gamma + \gamma')/\tau]^2 + g^2|\epsilon_0|^2/\tau^2} \\ &= \frac{i}{\tau} \int_{-x+i(\gamma+\gamma')/\tau}^{x+i(\gamma+\gamma')/\tau} \frac{ZdZ}{Z^2 + g^2|\epsilon_0|^2/\tau^2} \end{aligned} \quad (\text{A3})$$

Integrating Eq. (A3), we get

$$\text{Eq. (A3)} = \begin{cases} 0 & (> \text{pump threshold}) \\ \pi/\tau & (\leq \text{pump threshold}) \end{cases}$$

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